

Report of  
**Group Report**

Junior Hausdorff Trimester Program

**Kinetic Theory**

*Phase transitions, long-time behaviour and variational formulation of kinetic equations  
and  
Kinetic Theory of Growth Processes*

## Group members

**Junior members:** Marco Bonacini; Emre Esenturk; Antonio Esposito; Rishabh Gvalani; André Schlichting; Markus Schmidtchen

**Senior members:** Philippe Laurencot; Robert Pego, his PhD student Truong-Son Van

## Personal feedback (selection)

**Antonio Esposito:** Overall, my stay in Bonn and at HIM was fantastic. The organisation of the program was perfect as well as working conditions. The offices and the equipment were adequate. The staff of HIM was always ready to provide us support, even regarding our accommodation in Bonn. The success of program was possible thanks to the unique atmosphere of HIM.

**Philippe Laurencot:** The working conditions were almost perfect with a comfortable and well-located office, nearby printers, generous supply of coffee and cakes, and the administrative staff prior to my coming and during my stay was very efficient. The only drawback was the Internet access which was fluctuating in the building where my office was, whatever access I used, eduroam or the one provided by the center. As for the housing, it was simply great, well furnished and well located. I really enjoyed my stay in Bonn and would be eager to be invited again!

**Robert Pego:** The housing I had was excellent, and in a very convenient location. The design and environment of the Institute was very good indeed for encouraging discussion and collaboration. I recall only a couple of little things that caused any difficulties– heat on the top floor during the hottest days in June and July, and a weak wireless signal in the top floor office where my desk was.

**Markus Schmidtchen:** Staying at the HIM was truly rewarding. The atmosphere was collegial and sociable. In summary, the workshop was extremely enjoyable and worthwhile.

## Organized events

- Our group was part in organizing the workshop *Probabilistic and variational methods in kinetic theory*, which had in the very first week of the program the role of a kickoff event for the trimester. We as a group got many inputs, ideas and inspirations from the talks for the following research.
- Our group organized the lecture series by Martin Evans and Stefan Grosskinsky on stochastic models for condensation phenomena.
- Our group co-organized and actively participated in the Trimester seminar series on *coagulation-fragmentation*.

## Research output

- **Marco Bonacini, Barbara Niethammer and Juan Velazquez** investigated in [BNV19a, BNV19b] the stability of a special class of solutions to a coagulation-fragmentation equation with coagulation kernels close to the diagonal kernel and diagonal fragmentation kernels.
- Marco Di Francesco, **Antonio Esposito**, and **Markus Schmidtchen** obtained in [DFES20] the many-particle limit for a system of interaction equations driven by Newtonian potentials.
- Constantin Eichenberg and **André Schlichting** proved in [ES21] rigorously the coarsening behavior towards the self-similar profile for the exchange-driven growth process with product kernels.
- **Antonio Esposito**, Francesco S. Patacchini, **André Schlichting**, and Dejan Slepčev introduced in [EPSS21] a variational description of aggregation equations on graphs and general continuous nonlocal state spaces.
- **Emre Esenturk** and **Juan Velazquez** proved in [EV21] the longtime behavior of the exchange-driven growth model for kernels satisfying either a detailed balance condition or specific monotonicity assumptions.
- **Rishabh Gvalani** and **André Schlichting** obtained in [GS20] a mountain pass theorem in the space of probability measures with applications to metastability of stochastic interacting particle systems.
- **Philippe Laurencot** obtained in [Lau20] inspired by discussions with the participants Marina Ferreira and Juan J.L. Velazquez a characterization of stationary solutions to Smoluchowski's coagulation equation with sources.
- Hiroyoshi Mitake, Hung V. Tran and **Truong-Son Van** connected in [TV20, MTV20] a critical coagulation-fragmentation equation with multiplicative coagulation kernel and constant fragmentation kernel to the study of viscosity solutions to a new singular Hamilton-Jacobi equation. They proved wellposedness, regularity and long-time behaviors of viscosity solutions to the Hamilton-Jacobi equation in certain regimes, which have implications to wellposedness and long-time behaviors of *mass-conserving* solutions to the coagulation-fragmentation equation.

## Collaborations and works in progress

- B. Niethammer, R. Pego, A. Schlichting, and J. Velazquez** investigated a bubbleator model, which consists of a cluster growth model with input of small droplets and decay of large clusters. They obtained a formal limit model of the form of a hyperbolic equation together with a nonlocal boundary condition at the origin and proved rigorously the occurrence of a supercritical Hopf bifurcation for this model.
- A. Esposito, R. Gvalani, A. Schlichting, and M. Schmidtchen** worked during the time of the trimester on the variational description of the aggregation and homogeneous inelastic Boltzmann equation. They found a promising and novel nonlocal gradient structures providing a variational description for both equations as gradient flows of the kinetic energy.
- Ph. Laurencot** started several works still in progress dealing in spirit of [Lau20] with stationary solutions to coagulation-fragmentation equations for general coefficients, self-similar solutions for coagulation-fragmentation equations with specific choices of coagulation and fragmentation coefficients, and the non-existence issue for the exchange driven model.

**M. Ferreira, R. Pego, J. Velazquez** initiated a joint project on coagulation-fragmentation dynamics with sources, which will be pursued further when time and funding permit.

## References

- [BNV19a] Marco Bonacini, Barbara Niethammer, and Juan Velázquez. Solutions with peaks for a coagulation-fragmentation equation. Part I: stability of the tails. *arXiv:1906.08965*, June 2019.
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- [EPSS21] Antonio Esposito, Francesco Patacchini, André Schlichting, and Dejan Slepčev. The nonlocal interaction equation on graphs as gradient flow with respect to the upwind transportation metric. *submitted to Arch. Ration. Mech. Anal.*, 12 2021.
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- [GS20] Rishabh S. Gvalani and André Schlichting. Barriers of the McKean-Vlasov energy via a mountain pass theorem in the space of probability measures. *J. Funct. Anal.*, 279(11):108720, 34, 2020.
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- [TV20] Hung V. Tran and Truong-Son Van. Coagulation-Fragmentation equations with multiplicative coagulation kernel and constant fragmentation kernel. *accepted to Communications on Pure and Applied Mathematics, arXiv:1910.13424*, July 2020.