

## FINAL REPORT

### JUNIOR TRIMESTER PROGRAM KINETIC THEORY, HIM BONN

ESTHER S. DAUS

- **Group name:**  
Stochastic-PDE Bridge
- **Group members:**  
Julio Backhoff, Esther S. Daus, Helge Dietert, Giacomo Di Gesu, Amit Einav, Josephine Evans, Beatrice Signorello, Bao Q. Tang, Tobias Wöhrer, Junjian Yang.
- **Research interests:**  
Diffusion equations in kinetic theory: degenerate diffusion, hypocoercivity, cross-diffusion.  
Links between ideas from stochastic analysis and kinetic theory.

The group acknowledges the excellent working conditions provided by the Hausdorff Research Institute for Mathematics (HIM) and would also like to thank the staff at the HIM for their great help and support. Their excellent handling of the administrative tasks contributed significantly to the research productivity of the group. The group also highly appreciated the daily tea and cake breaks, which could be used to discuss mathematics and to socialize with other participants of the program. The outstanding funding and working environment for visitors afforded by the trimester program greatly helped our collaborations.

#### SCIENTIFIC ACTIVITIES

We are particularly grateful for the funding provided to organize the workshop *Probabilistic and Variational Methods in Kinetic Theory*, May 13 - 17, 2019, organized by Esther S. Daus, Giacomo Di Gesu, André Schlichting.<sup>1</sup>

- **Workshop name:** Probabilistic and Variational Methods in Kinetic Theory.
- **Organizers:** Esther S. Daus, Giacomo Di Gesu, André Schlichting.
- **Speakers (25):** Anton Arnold, Giada Basile, Martin Burger, Li Chen, Matias Delgadino, Laurent Desvillettes, Amit Einav, Matthias Erbar, Antonio Esposito, Jo Evans, Simone Fagioli, Joaquin Fontbona, Giambattista Giacomini, Rishabh Gvalani, Franca Hoffmann, Ansgar Jüngel, Shi Jin, Jean-Christophe Mourrat, Francesco Patacchini, Mario Pulvirenti, Angela Stevens, Bao Q. Tang, Hagop Tossounian, Oliver Tse, Boguslaw Zegarlinski.
- **Aim of the workshop:** The workshop aimed to bring the communities from the field of stochastic processes and partial differential equations related to kinetic models together. These include for instance the dynamic behavior of kinetic Fokker-Planck and Vlasov-McKean equation with linear or nonlinear (cross-)diffusion. The workshop aimed to highlight recent advances on hypocoercivity, the rate of convergence, self-similarity, and phase transitions. For this, ideas from the side of stochastic analysis, as well as the variational formulation of gradient flows yielded exciting results in kinetic theory. The workshop aimed to deepen these links and build new bridges. Many of the talks were recorded by the HIM and are available online.

The workshop organized by our group during this trimester was very instructive, as it allowed for a great group of researchers from different areas but common interests to meet and exchange perspectives. It was interesting to see how people from Analysis and Geometry were interested in similar questions as people in the field of Applied Probability, and how they deployed completely different tools to tackle related questions.

#### LIST OF SOME MATHEMATICAL DISCUSSIONS DURING THE TRIMESTER PROGRAM

Below we give an overview of the progress made by the researchers in our group that was facilitated by the HIM trimester program.

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<sup>1</sup><https://www.him.uni-bonn.de/programs/past-programs/past-junior-trimester-programs/kinetic-theory-2019/workshop-probabilistic-and-variational-methods-in-kinetic-theory-may-13-17-2019/>

- Bao Quoc Tang discussed with Amit Einav a method for indirect diffusion effects to reaction-diffusion systems, *i.e.* an effective diffusion for the non-diffusive species which is incurred by a combination of diffusion from diffusive species and reversible reactions between the species. This led to the publication [5].
- Amit Einav together with Beatrice Signorello, Tobias Wöhrer and Anton Arnold started the research into the notion of the generalised Fisher Information in defective (but not degenerate) Fokker-Planck equations and its relevance to the sharp convergence to equilibrium rate. They finished [1], in which they study the Goldstein-Taylor equations (a simplified version of a BGK system), and provide a general method to tackle the question of convergence to equilibrium when the relaxation function is not constant in a quantitative way.
- Bao Quoc Tang discussed with Josephine Evans and Dietert Helge on possible connections between indirect diffusion effects and hypocoercivity.
- Julio Backhoff and Junjian Yang started a cooperation on the subject of non-exponential large deviations and Laplace principles for martingales.
- Julio Backhoff and Giacomo Di Gesù had discussions concerning non-standard large deviation principles for Donsker-type approximation schemes.
- Helge Dietert and Esther S. Daus had discussions with André Schlichting and Matthias Erbar about cross diffusion and gradient flows.
- Helge Dietert finished some works on nonlocal cross diffusion and its entropy structure in [4].
- After several talks about hypocoercivity, Helge Dietert started to discuss with Josephine Evans what is the best (spatially-dependent) noise for a linear kinetic model so that the decay rate is as fast as possible.
- Giacomo Di Gesù had discussions with F. Hérau on tunnelling effects for Kinetic Fokker-Planck equations, and with M. Ottobre on long-time behaviour for infinite-dimensional hypoelliptic equations.
- Jo Evans had an ongoing project with Havva Yoldas about applying probabilistic hypocoercivity methods to run and tumble chemotaxis equations. Jo Evans finished together with Ivan Moyano their preprint [6].
- Esther S. Daus had discussions with S. Jin and L. Liu about mathematical problems related to uncertainty quantification which led to [3], where they studied the nonlinear multi-species Boltzmann equation with random uncertainty coming from the initial data and collision kernel. Moreover, Esther S. Daus discussed with Junjian Yang about mean-field limits and Cucker-Smale flocking models, and with Alexandra Holzinger about the derivation of Shigesda-Kawaskai-Teramoto type cross-diffusion systems from an interacting particle system, which was recently finished in [2].

#### FOLLOW-UP EVENTS ORGANIZED BY GROUP MEMBERS IN THE SPIRIT OF THE JUNIOR TRIMESTER PROGRAM

- Julio Backhoff together with some colleagues will be organizing a CIRM conference in November 2021 on the subject of mean-field control, games, and the Schrödinger problem.
- Esther S. Daus together with some colleagues will be organizing the Online Hausdorff School *Diffusive Systems* at the Hausdorff School of Advanced Studies in Mathematics (HSM) Bonn, April 12-16, 2021.

#### RESEARCH OUTPUT: COMPLETED PREPRINTS/PAPERS THAT AROSE FROM THE TRIMESTER PROGRAM

##### REFERENCES

- [1] A. Arnold, A. Einav, B. Signorello and T. Wöhrer. *Large time convergence of the non-homogeneous Goldstein-Taylor equation*. To appear in J. Stat. Phys. arXiv:2007.11792.
- [2] L. Chen, E. S. Daus, A. Holzinger, A. Jüngel. *Rigorous derivation of population cross-diffusion systems from moderately interacting particle systems*. Submitted for publication, 2020, arXiv:2010.12389.
- [3] E. S. Daus, S. Jin, L. Liu: *On the multi-species Boltzmann equation with uncertainty and its stochastic Galerkin approximation*. Submitted for publication, 2019, arXiv:1909.01231.
- [4] H. Dietert, A. Moussa. *Persisting entropy structure for nonlocal cross-diffusion systems*. Submitted for publication (2021), arXiv:2101.02893.
- [5] A. Einav, J. Morgan, Bao Q. Tang. *Indirect diffusion effect in degenerate reaction-diffusion systems*. SIAM Journal of Mathematical Analysis. Vol. 52 (5) (2020) 4314-4361.
- [6] J. Evans, I. Moyano. *Quantitative Rates of Convergence to Equilibrium for the Degenerate Linear Boltzmann Equation on the Torus*. Submitted for publication, 2020, arXiv:1907.12836.