FINAL REPORT

Study of a fractional kinetic Fokker-Planck equation

Junior Trimester Program in Kinetic Theory

Group members: Nathalie Ayi, Maxime Breden, Maxime Herda, Hélène Hivert.

Research interests: Fractional Laplacian, Fokker-Planck operator, fractional diffusion, heavy-tailed distribution, (linear) kinetic equations, numerical method, hypocoercivity.

Scientific Activities

The group is grateful for the opportunity to be part of this Junior Trimester Program in Kinetic Theory and warmly acknowledge the HIM for the financial support and the hospitality they benefited during their stay. We also are particularly grateful for the funding provided to organise the workshop below.

Workshop: Qualitative behaviour of kinetic equations and related problems: numerical and theoretical aspects

This workshop focused on qualitative analysis of problems arising in Kinetic theory, both from theoretical and numerical viewpoints. We explored various mathematical directions that are useful to obtain relevant information on the behavior of the solutions, including questions as various as well-posedness and regularity, numerical simulation, long-time or asymptotic behaviour (scale limits, homogenization, asymptotic and long-time preserving schemes) and controllability. The workshop was a very nice event with about 40 participants and 18 talks.

Organisers: Nathalie Ayi, Ariane Trescases, Harsha Hutridurga Ramaiah.

Speakers: Marianne Bessemoulin-Chatard, Maxime Breden, Carlos Castro, Ludovic Cesbron, Anais Crestetto, Helge Dietert, Megan Griffin-Pickering, Jessica Guérand, Frédéric Hérau, Ning Jiang, Laurent Laflèche, Donghyun Lee, Pierre Monmarché, Arnaud Munch, Karel Pravda-Starov, Miguel Rodrigues, Francesco Salvarani, Havva Yoldas.

Research outputs

- **Nathalie Ayi**, **Maxime Herda** and **Hélène Hivert** acknowledge fruitful discussions with Markus Schmidtchen about the analysis of the long time behavior and entropy structure of Fokker-Planck type equations.

- **Nathalie Ayi** acknowledges fruitful discussions with Raphael Winter and Juan Velázquez about discrete velocity models and the derivation of mesoscopic models starting from the microscopic ones.

- **Maxime Breden** and **Maxime Herda** initiated discussions with Ariane Trescases on the derivation of nonlinear parabolic systems of PDEs with cross-diffusion from kinetic systems of PDEs. Together, they proposed a new kinetic description of particle dynamics with velocities depending on the particle density. From this mesoscopic model, they showed that in the appropriate diffusive limit, velocity averages of the distribution function converge to a solution of the macroscopic model. This work is in progress and will be concluded in a forthcoming preprint [BHT21].

- **Hélène Hivert** met Havva Yoldaş during the Junior Trimester Programm "Kinetic theory" and she proposed her to continue to work together. Havva Yoldaş has been proposed a postdoc position at ICJ, Lyon (France). They worked on asymptotic preserving schemes, for selection-mutation equations arising in population dynamics. In a regime of long time and small mutations, the solution of this equation converges to the viscosity solution of a constrained Hamilton-Jacobi equation. The scheme they proposed enjoys stability properties in the transition to the asymptotic regime. In particular, the viscosity solution of the limit problem is well approximated by the asymptotic scheme. They discussed the construction of the scheme and proved of its properties.

- **Nathalie Ayi**, **Maxime Herda**, **Hélène Hivert** and Isabelle Tristani work on the collaborative research project that was initiated for the Junior Trimester Program in Kinetic Theory at the HIM. It leads to two papers.
  In the first one [AHHT20], we are interested in the large time behavior of linear kinetic equations with heavy-tailed local equilibria. Our main contribution concerns the kinetic Lévy-Fokker- Planck equation, for which we adapt hypocoercivity techniques in order to show that solutions converge exponentially fast to the global equilibrium. Compared to the classical kinetic Fokker-Planck equation, the issues here concern the lack of symmetry of the non-local Lévy-Fokker- Planck operator and the understanding of its regularization properties. As a complementary related result, we also treat the case of the heavy-tailed BGK equation. This paper has been published in *Comptes Rendus. Mathématique.*
  In the second one [AHHT21], we introduce and analyse numerical schemes for the homogeneous and the kinetic Léevy-Fokker-Planck equation. The discretizations are designed to preserve the main features of the continuous model such as conservation of mass, heavy-tailed equilibrium and (hypo)coercivity properties. We perform a thorough analysis of the numerical scheme and show exponential stability. Along the way, we introduce new tools of discrete functional analysis, such as discrete nonlocal Poincaré and interpolation inequalities adapted to fractional diffusion. Our theoretical findings are illustrated and complemented with numerical simulations.

- **Nathalie Ayi** and **Hélène Hivert** initiated discussions about radiative transfer type equations with heavy-tailed equilibrium. In a work in progress, they consider anomalous diffusion limits and numerical considerations associated with it.

References

[AHHT20] N. Ayi, M. Herda, H. Hivert, and I. Tristani. A note on hypocoercivity for kinetic equations with heavy-tailed equilibrium. *Comptes Rendus. Mathématique*, 358(3):333–340, 2020.

[AHHT21] N. Ayi, M. Herda, H. Hivert, and I. Tristani. On a structure-preserving numerical method for fractional Fokker-Planck equations. Submitted preprint, July 2021.

[BHT21] M. Breden, M. Herda, and A. Trescases. On the derivation of nonlinear parabolic systems as diffusive limits of systems of kinetic equations with heterogeneous velocities. Preprint in preparation, 2021.