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Workshop on

"Effective equations: frontiers in classical and quantum systems"

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organized by

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Abstracts

Alexander Bobylev (Keldysh Institute of Applied Mathematics, Russian Academy of Science, Moscow)

On some properties of Vlasov-Poisson-Landau kinetic equations

Abstract: The talk is related to mathematical problems of kinetic theory of plasma. We consider some properties of the Vlasov- Poisson-Landau kinetic equations for the case when the typical length is the mean free path, not the Debye radius. What are the limiting equations in that case? Are they well posed? How to describe the corresponding necessary conditions of transition to the limit? These and similar questions are the main subject of the talk. All our formal considerations for the non-linear case are supported by more serious analysis of the linearized equations and their exact solutions. The talk is based on recently published paper: Bobylev A.V. and Potapenko I.F., Long wave asymptotics for Vlasov-Poisson-Landau kinetic equation, J.Statist. Phys., 175 (2019), pp. 1-18.

Eric A. Carlen (Rutgers University)

A Kac Type Master Equation with Exclusion

Abstract: The investigation of Kac type master equations with exclusion was initiated in a paper of Colangeli, Pezzotti and Pulvirienti. They introduced a partition of the particle phase space into cubic cells of equal volume and suppressed collisions that led to multiple occupancy. From this "classical" model with exclusion, they derived the spatially homogeneous Uehling-Uhlenbeck equation for Fermions. We consider a related model with exclusion that prevents particles from occupying states that are too close, but without a preordained partition of phase space. We investigate the stationary states, the rate of convergence to them, propagation of chaos and the a kinetic equation for the evolution of the empirical distribution. This is joint work with Bernt Wennberg.

Maria Carvalho (Rutgers University)

Spectral Gaps and Entropy Production for Kinetic Master Equations with Degenerate Jump Rates

Abstract: The original Kac model concerned Maxwellian molecules, for which the collision rate does not depend on the magnitude of the relative velocity of the colliding particles This model is now well-understood at both the level of spectral gaps and entropy production. More physically realistic models, especially for hard sphere collisions, still pose significant challenges. In recent work with Carlen and Loss, the Kac conjecture for the spectral gap was proved for hard sphere collisions, and in fact for the full range of collision models all the way from Maxwellian to Super-Hard Spheres. The investigation of Super-Hard Sphere collisions was initiated by Villani, who showed that "speeding up" the rate of high energy collisions led to the strong entropy production inequalities that Cercignani had conjectured. However, his result depends on suppressing the degeneracy at low energies. It is natural to conjecture, that as with the spectral gap, entropy production bounds of Cercignani type hold for super hard sphere collisions without suppressing the low energy degeneracy.

Laurent Desvillettes (Université Paris Diderot, IMJ-PRG)

From Boltzmann equations for mixtures towards thin spray equations

Abstract: Thin spray equations consist in the coupling (through the drag force) of a kinetic (Vlasov) equation and a fluid equation. We explain how those equations can be recovered (at least at the formal level) from systems of Boltzmann equations for mixtures under suitable scalings (involving the Knudsen number and the relative mass of the particles involved). The results which will be presented were obtained with François Golse and Valeria Ricci.

Miguel Escobedo Martínez (Universidad del País Vasco)

Some remarks on the three waves collision integral+condensate.

Abstract: Some results will be first presented on the the solutions of a coupled system of nonlinear equations that describe the interactions in a spatially homogeneous and isotropic gas of bosons between the normal fluid particles and the condensate near the critical temperature. Special attention will be given to the behavior near zero of the density of the particles in the normal fluid.

Francis Filbet (Institut de Mathématiques de Toulouse)

Asymptotics of the three dimensional Vlasov equation in the large magnetic field limit

Abstract: We study the asymptotic behavior of solutions to the Vlasov equation in the presence of a strong external magnetic field. In particular we provide a mathematically rigorous derivation of the guiding-center approximation in the general three dimensional setting under the action of large inhomogeneous magnetic fields. First order corrections are computed and justified as well, including electric cross field, magnetic gradient and magnetic curvature drifts. We also treat long time behaviors on two specific examples, the two dimensional case in cartesian coordinates and a poloidal axisymmetric geometry, the former for expository purposes. Algebraic manipulations that underlie concrete computations make the most of the linearity of the stiffest part of the system of characteristics instead of relying on any particular variational structure.

Irene M. Gamba (The University of Texas at Austin)

The Cauchy problem and BEC stability for the quantum Boltzmann-Condensation System at very low temperature

Abstract: After a short review of classical kinetic collisional theory from a probabilistic viewpoint, we discuss a new model for a coupled quantum Boltzmann-Condensation system that describes the evolution of the interaction between a well formed Bose-Einstein Condensate (BEC) and the quasi-particles cloud. The kinetic part of the model, derived as weak turbulence kinetic model from a quantum Hamiltonian, is valid for a dilute regime at which the temperature of a bosonic gas is very low compared to the Bose-Einstein condensation critical temperature. In particular, the system couples the density of the condensate from a Gross-Pitaevskii type equation to the kinetic equation through the dispersion relation in the kinetic model and the corresponding transition probability rate from pre to post collision momentum states.

We show the well-posedness of the Cauchy problem to the system for bounded solutions with a sufficiently large number of statistical moments, find qualitative properties of the solution such as instantaneous creation of exponential tails, and prove the condensate uniform stability related to the initial mass ratio between condensed particles and quasi-particles. This stability result leads to global in time existence of bounded, finite energy solutions to an initial value problem for the quantum Boltzmann-Condensation system.

This is work in collaboration with Ricardo J. Alonso and Minh Binh Tran

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François Golse (Ecole polytechnique, Centre de Mathématiques Laurent Schwartz)

Time-Splitting Algorithms for Quantum Dynamics in the Semiclassical Regime

Abstract: We establish a new type of convergence rate estimates for time-splitting numerical schemes applied to the von Neuman equation governing the evolution of density operators in quantum dynamics. These estimates are uniform in the Planck constant, so that time-splitting strategies can be used with large time-steps even in the semiclassical regime. Our analysis is based on the construction of a functional measuring the distance between a quantum density operator and a probability density in the classical phase space that is reminiscent of the Monge-Kantorovich-Wasserstein distance used in optimal transport. (Joint work with Shi Jin and Thierry Paul)

Richard Höfer (University of Bonn)

On the sedimentation of particle suspensions in Stokes flows

Abstract: Small particles moving in a fluid are encountered in various situations in nature and technology. In many cases, gravitation is the driving force for the dynamics of the particles. If the particles are not too small, the system can be microscopically modeled by the Navier-Stokes equations coupled with a system of ODEs for the particles according to Newton's laws. Although the force acting on each particle due to the gravity is directly proportional to its mass, and we do not include direct (e.g. electromagnetic) interaction between the particles themselves, the motion of the particles will be quite complex in many situations. The complexity arises from the interaction of the particles through the fluid. Indeed, the presence of each particle induces a disturbance in the fluid flow which again influences all the other particles. Assuming that the fluid inertia is negligible, we will present different microscopic models for spherical and non-spherical particles with and without inertia. We will then discuss corresponding macroscopic models which consist of systems that couple a Vlasov equation to Stokes equations. In the case of inertialess spherical particles, the macroscopic system can be rigorously derived from the microscopic dynamics in the limit of many small particles.

Jens Marklof (University of Bristol)

Kinetic transport in the Lorentz gas: classical and quantum

Abstract: In the first part of this lecture, I will discuss the proof of convergence of the Lorentz process, in the Boltzmann-Grad limit, to a random process governed by a generalized linear Boltzmann equation. This will hold for general scatterer configurations, including certain types of quasicrystals, and include the previously known cases of periodic and Poisson random scatterer configurations. The second part of the lecture will focus on quantum transport in the periodic Lorentz gas in the same Boltzmann-Grad limit, and I will report on some partial progress in this challenging problem. Based on joint work with Andreas Strombergsson (part I) and Jory Griffin (part II).

Robert I. A. Patterson (Weierstrass Institute, Berlin)

Interaction Clusters for the Kac Process

Abstract: Interaction clusters form closed dynamical systems in the billiard model for an ideal gas, which is hard to analyse. Kac introduced a more tractable, stochastic model for an ideal gas and in this setting I will explain what interaction clusters are and show why they satisfy a generalisation of the Smoluchowski coagulation equation. I will also present a representation of the clusters as connected components of an inhomogeneous random graph and show that they undergo a phase transition at a critical time given in terms of the norm of a convolution operator on a space of square integrable functions. This is joint work with Wolfgang Wagner, Sergio Simonella and Daniel Heydecker.

Nataša Pavlović (The University of Texas at Austin)

From a classical system of particles to a ternary Boltzmann equation

Abstract: In this talk we present a rigorous derivation of a new kinetic equation describing the limiting behavior of a classical system of particles with three particle instantaneous interactions. The equation, which we call ternary Boltzmann equation, can be understood as a step towards modeling a dense gas in non-equilibrium. It is derived from laws of instantaneous three particle interactions, preserving momentum and energy. The talk is based on a joint work with Ioakeim Ampatzoglou.

Peter Pickl (LMU Munich)

Derivation of the Vlasov equation

Abstract: The rigorous derivation of the Vlasov equation from Newtonian mechanics of N Coulomb-interacting particles is still an open problem. In the talk I will present recent results, where an N-dependent cutoff is used to make the derivation possible. The cutoff is removed as the particle number goes to infinity. Our result holds for typical initial conditions, only. This is, however, not a technical assumption: one can in fact prove deviation from the Vlasov equation for special initial conditions for the system we consider. Joint work with Niklas Boers Phillip Grass and Dustin Lazarovici.

Marcello Porta (University of Tübingen)

On the correlation energy of interacting fermionic systems in the mean-field regime

Abstract: In this talk I will discuss the ground state properties of a homogeneous, interacting Fermi gas, in the mean-field regime. In particular, I will focus on the correlation energy, defined as the difference between many-body and Hartree-Fock ground state energies. Recently, progress has been made in the rigorous understanding of the precise value of this quantity. I will present an upper bound for the correlation energy that agrees with an old prediction of Gell-Mann and Brueckner in the physics literature, whose proof is based on a rigorous bosonization technique. Then, I will discuss a lower bound, matching the upper bound for small interaction potentials.
The upper bound is based on a joint work with N. Benedikter, P. T. Nam, B. Schlein and R. Seiringer, while the lower bound is based on a joint work with C. Hainzl and F. Rexze.

Mario Pulvirenti (Sapienza, University of Rome)

Propagation of chaos for exotic interactions

Abstract: We consider a N-particle model describing an alignment mechanism due to a topological interaction among the agents. We show that the kinetic equation, expected to hold in the mean-field limit, can be rigorously derived. This means that the statistical independence is recovered in the limit, provided it is assumed at time zero.

Benjamin Schlein (University of Zurich)

Excitation spectrum and time-evolution of Bose-Einstein condensates

Abstract: We consider systems of N trapped bosons interacting through a potential with scattering length of the order 1/N (Gross-Pitaevskii regime). We present recent tools that can be used to describe the time-evolution resulting from a change of the external fields and also to determine the low-energy excitation spectrum in the limit of large N. This talk is based on joint work with C. Boccato, C. Brennecke and S. Cenatiempo.

Herbert Spohn (Technische Universität München)

Hydrodynamics of the classical Toda chain

Abstract: Over the past four years there have many activities, mostly on the quantum side, to study the dynamics of integrable many-particle systems at non-zero temperatures. Apparently they share common features. The classical Toda chain will serve as a road map in illustrating the recent advances.

Robert M. Strain (University of Pennsylvania)

Global mild solutions of the Landau and non-cutoff Boltzmann equation

Abstract: This paper proves the existence of small-amplitude global-in-time unique mild solutions to both the Landau equation including the Coulomb potential and the Boltzmann equation without angular cutoff. Since the well-known works (Guo, 2002) and (Gressman-Strain-2011, AMUXY-2012) on the construction of classical solutions in smooth Sobolev spaces which in particular are regular

in the spatial variables, has still remained an open problem to obtain global solutions in an $L^\infty_{x,v}$ framework, similar to that in (Guo-2010), for the Boltzmann equation with cutoff in general bounded domains. One main difficulty arises from the interaction between the transport operator and the velocity-diffusion-type collision operator in the non-cutoff Boltzmann and Landau equations; another major difficulty is the potential formation of singularities for solutions to the boundary value problem. In this work we introduce a new function space with low regularity in the spatial variable to treat the problem in cases when the spatial domain is either a torus, or a finite channel with boundary. For the latter case, either the inflow boundary condition or the specular reflection boundary condition is considered. An important property of the function space is that the $L^\infty_T L^2_v$ norm, in velocity and time, of the distribution function is in the Wiener algebra $A(\Omega)$ in the spatial variables. Besides the construction of global solutions in these function spaces, we additionally study the large-time behavior of solutions for both hard and soft potentials, and we further justify the property of propagation of regularity of solutions in the spatial variables. To the best of our knowledge these results may be the first ones to provide an elementary understanding of the existence theories for the Landau or non-cutoff Boltzmann equations in the situation where the spatial domain has a physical boundary. This is a joint work with Renjun Duan (The Chinese University of Hong Kong), Shuangqian Liu (Jinan University) and Shota Sakamoto (Tohoku University).
