Stochastic and kinetic models of condensation

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Abstract

We review recent and classical results on aggregation/clustering models on a microscopic scale in the framework of stochastic particle systems, and on a mesoscopic scale using kinetic equations for discrete mass distributions. Both approaches are used to describe phenomena such as condensation, (instantaneous) gelation and coarsening, and we show how they are related in a mean-field scaling limit of stochastic particle systems. We also discuss differences between both descriptions such as volume constraints, and introduce size-biased versions of the dynamics to study the coarsening behaviour for condensing systems with bounded activity. In the last lecture we focus models with product aggregation kernels and unbounded activity, and summarize results for condensation dynamics and instantaneous condensation. Examples that will be used as illustrations throughout include the zero-range process, the inclusion process and generalizations, and Becker-Döring models.

Since my background is in probability rather than kinetic theory, I naturally will have a bit more to say about particle systems than kinetic models. Part of the lecture is based on recent work with Watthanan Jatuviriyapornchai (Bangkok) and Paul Chleboun (Warwick).

Literature overview

Spatially homogeneous SPS

- introduction of SPS in the mathematics literature including exclusion process (EP) and zero-range (ZRP) [Spitzer, 1970]
  construction on infinite lattices [Holley, 1970]
  graphical construction [Harris, 1972]

- zero-range process (ZRP) [Liggett, 1973]
  construction on infinite lattice and stationary product measures [Andjel, 1982]
  generalized class of misanthrope processes[Cocozza-Thivent, 1985]
  existence of dynamics on infinite lattices for ZRP and related models with superlinear jump rates[Balázs et al., 2007]
- mass transport models including continuous mass and multi-particle jumps [Zia et al., 2004, Evans et al., 2004b]

multiple particles moving in ZRP [Greenblatt and Lebowitz, 2006]
several species mass transport [Hanney, 2006]
mass migration models [Fajfrová et al., 2016]

- inclusion process [Giardinà et al., 2007, Giardinà et al., 2009, Giardinà et al., 2010, Carinci et al., 2013]

Condensation in homogeneous SPS

- backgammon model [Bialas et al., 1997], simple urn model [Drouffe et al., 1998], bus route model[O'Loan et al., 1998]
  ZRP [Evans, 2000, Godrèche, 2003], [Godrèche, 2019]
  reviews [Evans and Hanney, 2005], [Godrèche, 2007]
- finite number of lattice sites [Ferrari et al., 2007], monotonicity [Rafferty et al., 2017]
  condensation in IP [Grosskinsky et al., 2011, Grosskinsky et al., 2013]
- instability of the condensation transition [Jeon, 2010], due to quenched random interaction [Grosskinsky et al., 2008, del Molino et al., 2012]
- thermodynamic limit of ZRP invariant measures
  [Jeon et al., 2000, Jeon and March, 2000], equivalence of ensembles, LLN for condensate [Grosskinsky et al., 2003]
  CLT for condensate [Armendáriz and Loulakis, 2009]
  conditioning on continuous mass [Armendáriz and Loulakis, 2011]
- at criticality [Armendáriz et al., 2013]
  finite size effects [Chleboun and Grosskinsky, 2010, Juntunen et al., 2010]
  extreme value statistics [Evans and Majumdar, 2008], canonical analysis [Evans et al., 2004b], [Majumdar et al., 2005, Evans et al., 2006b]
- discontinuous condensation transition for size-dependent ZRP [Grosskinsky and Schütz, 2008, Chleboun and Grosskinsky, 2015], non-equivalence of ensembles [Touchette, 2009]
  non-equivalence from super-extensive mass in ZRP [Gradenigo and Bertin, 2017]
  hierarchical structure of the condensed phase for IP condensation [Chleboun et al., 2019]
- beyond product measures
  target process [Luck and Godrèche, 2007]
  chipping model [Rajesh and Majumdar, 2001]
  pair-factorized steady states [Evans et al., 2006a], [Waclaw et al., 2009a, Waclaw et al., 2009b] [Ehrenpreis et al., 2014, Chatterjee et al., 2015]
  open boundary [Nagel et al., 2015, Nagel and Janke, 2016]
  different left-right dynamics in ZRP [Chatterjee and Mohanty, 2017]
- several conservation laws:
  two species [Evans and Hanney, 2003, Hanney and Evans, 2004], [Grokinsky, 2008], stationary distributions [Grosskinsky and Spohn, 2003]
  inducing condensation [Szavits-Nossan et al., 2014b, Szavits-Nossan et al., 2014a, Szavits-Nossan et al., 2016]

- explosive condensation models [Evans and Waclaw, 2014, Waclaw and Evans, 2012]
  reviews beyond ZRP models [Chleboun and Grosskinsky, 2014, Evans and Waclaw, 2015]

Dynamics of homogeneous condensation in SPS

- heuristics on coarsening dynamics in explosive condensation models:
  asymmetric 1D [Waclaw and Evans, 2012], symmetric 1D symmetric [Chau et al., 2015],
- hydrodynamic limits for non-condensing SPS [Kipnis and Landim, 2013]
  for condensing SPS heuristics [Schütz and Harris, 2007], rigorous result for subcritical profiles [Stamatakis, 2015]
- nucleation on finite lattices in IP [Grosskinsky et al., 2013], in ZRP [Beltrán et al., 2015]
  heuristics for IP on infinite lattice [Cao et al., 2014]
- coarsening dynamics
  in ZRP [Godrèche, 2003, Godrèche and Luck, 2005],
  [Godrèche and Drouffe, 2016, Jatuviriyapornchai and Grosskinsky, 2016]
  in IP on finite lattice[Grosskinsky et al., 2013], on infinite lattice heuristics [Cao et al., 2014]
  rigorous using duality [Carinci et al., 2017, Ayala et al., 2019]
- stationary dynamics/metastability
  ZRP reversible on finite lattices [Beltrán and Landim, 2012a]
  ZRP thermodynamic limit limit [Bovier and Neukirch, 2014, Armendáriz et al., 2015]
  IP on regular graphs [Grosskinsky et al., 2013]
  on general graphs with 3 time scales [Bianchi et al., 2017]
  asymmetric ZRP: totally asymmetric [Landim, 2014], partially asymmetric [Seo, 2019]
- rigorous results on mean field limits: ZRP fluid limit [Graham, 2009]
  mean field rate equations for misanthrope[Jatuviriyapornchai and Grosskinsky, 2017]

Cluster aggregation and fragmentation

- Deterministic and stochastic models for coalescence (aggregation and coagulation): a review of the mean-field theory for probabilists [Aldous, 1999]
  kinetics of aggregation, scaling theory review [Leyvraz, 2003]
- Becker-Döring model [Becker and Döring, 1935]
  solutions to Becker-Döring equations [Ball et al., 1986, Wattis and King, 1999]
  Fokker-Planck equation related to Becker-Döring [Conlon and Schlichting, 2019]
- Smoluchowski equation [Smoluchowski, 1917]
  gelation [van Dongen, 1987], proof of gelation [Jeon, 1998], regularization with cutoff [C Ball et al., 2011]
- Random coagulation, Marcus Lushnikov process
  conjecture on gelation with simulations [Spouge, 1985], proof [Jeon, 1999], exact results [Lushnikov, 2005]
- cluster growth driven by monomer exchange
  [Brilliantov and Krapivsky, 1991], [Majumdar et al., 1998], [Ben-Naim and Krapivsky, 2003],

spatial bias [Rajesh and Krishnamurthy, 2002]
rigorous work [Esenturk, 2018, Schlichting, 2018, Esenturk and Velazquez, 2019],
role of volume conservation [Esenturk and Connaughton, 2019]

Extensions to homogeneous SPS

- extension of the dynamics:
  moving condensate in ZRP with stretched exponential, all but one particle jump simultaneously [Whitehouse et al., 2014]
  non-Markov ZRP motion of condensate: [Hirschberg et al., 2009, Hirschberg et al., 2012]
  non-conserving ZRP [Angel et al., 2007], condensing ZRP with additional diffusion [Levine et al., 2004]
- ZRP conditioned on atypical current [Hirschberg et al., 2015]
- multiple condensates:
  [Schwarzkopf et al., 2008, Thompson et al., 2010]
  finite capacities for sites: [Gupta and Barma, 2015], [Ryabov, 2014]

Tools

- metastability:
  potential theoretic approach to metastability[Bovier and den Hollander, 2015]
  martingale approach [Beltrán and Landim, 2010, Beltrán and Landim, 2012b, Beltrán and Landim, 2015]
  developed for SPS and then applied to more general processes
  review on Markov chains with martingales [Landim, 2019]
  the trace process [Landim et al., 2018]
- macroscopic fluctuation theory [Bertini et al., 2015a]
  martingale methods, fluctuations [Komorowski et al., 2012] empirical flows: [Bertini et al., 2015b]
- decomposition of dynamics: [De Carlo and Gabrielli, 2017, Kaiser et al., 2018] using also gradient flow structure [Schlichting, 2018]
- mean field/propagation of chaos: [Pra, 2017], SEP [Rezakhanlou, 1994],
  discrete Boltzmann [Rezakhanlou, 1996]
  [Delarue et al., 2019], more papers to come by Ramanan et al.
- equivalence of ensembles
  relative entropy, information theory [Csiszar, 1975, Csiszar, 1984]
  convexity of relative entropy in [Georgii, 1988]
  Csiszár Körner [Csiszár and Körner, 2011]
  large deviation and statistical mechanics[Touchette, 2009]

Applications of condensation results for SPS

- mapping of ZRP to diffusive systems: phase separation in multi-species models [Kafri et al., 2002a, Evans et al., 2004a]
  powerful criterion managed to resolve controversy over phase separation [Kafri et al., 2002b, Kafri et al., 2003], see review [Schütz, 2003] for further references
- wealth condensation [Burda et al., 2002], traffic modelling [Kaupužs et al., 2005]
- condensation in population genetics [Dereich and Morters, 2013]
- ZRP for shaken granular gases [van der Meer et al., 2004, Török, 2005] and references therein
  urn models for separation of sand [Eggers, 1999, Lipowski and Droz, 2002]
- networks:
  network growth [Ferretti and Bianconi, 2008, Bianconi and Barabási, 2001] [Godrèche and Luck, 2010]
  phase transition in Markovian bipartite graphs [Pulkkinen and Merikoski, 2005]
  autocatalytic networks, generalizations of inclusion process [Hoessly and Mazza, 2019]

Spatially inhomogeneous models

- condensation in ZRP with random rates [Evans, 1996]
  rigorous results including coupling [Andjel et al., 2000, Ferrari and Sisko, 2007] [Chleboun and Grosskinsky, 2
- EP with random rates [Krug and Ferrari, 1996, Benjamini et al., 1996]
- interplay between interaction and geometry driven condensation in ZRP
  [Godrèche and Luck, 2012, Mailler et al., 2016] ZRP with a single defect site [Angel et al., 2004]
  condensing ZRP with open boundary [Levine et al., 2005]
- ZRP on networks:
  [Bogacz et al., 2007, Waclaw et al., 2007, Waclaw, Bartłomiej et al., 2008]
  complete condensation on scale-free networks [Noh et al., 2005, Noh, 2005]
  condensation on weighted scale-free networks [Tang et al., 2006]
  disordered urn models [Ohkubo, 2007]
- IP [Grosskinsky et al., 2011]
  dynamics of condensation [Bianchi et al., 2017]

Books/monographs

- Monographs on stochastic/interacint particle systems (SPS or IPS) [Liggett, 1985, Liggett, 2013, Liggett, 2010],
- hydrodynamic limits of SPS[Kipnis and Landim, 2013]
- fragmentation and coagulation processes [Bertoin, 2006]
- metastability [Bovier and den Hollander, 2015]
- fluctuations of Markov processes [Komorowski et al., 2012]

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